**YYEAR 11 MATHS SPECIALIST SEMESTER 2 2019**

**INVESTIGATION #3**

Time Allowed: 60 minutes Total Marks: 31

Throughout this investigation you are required to state conclusions with sufficient evidence to justify them.

When writing rules use notations such as: n + 1, **F**n, fn+1, **T**a+b, tn+2 etc

**PART A**

**Triangle Matrices**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| The triangle sequence is: | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 | 45 | 55 | … |
|  | t1 | t2 | t3 | t4 | t5 | t6 | t7 | t8 | t9 | t10 |  |

In this part you are going to explore triangle matrices.

If tn, tn+1, and tn+2 are consecutive triangle numbers then a triangle matrix is defined by

**T**n = 

The first five triangle matrices will be:



Using the above definitions, investigate the following. You will need to show some examples to support your conclusions.

**1.** The addition of two consecutive triangle matrices. [4 marks]

**2.** The value of the determinant of the triangle matrices [3 marks]

**3.** The squaring of triangle matrices [4 marks]

**4.** The product of two consecutive triangle matrices [4 marks]

**TOTAL 15 marks**

**PART B**

**Fibonacci Matrices**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| The Fibonacci sequence is: | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | … |
|  | f1 | f2 | f3 | f4 | f5 | f6 | f7 | f8 | f9 | f10 | f11 | f12 |  |

In this part you are going to explore Fibonacci matrices.

If fn, fn+1, and fn+2 are consecutive Fibonacci numbers then a Fibonacci matrix is defined by

**F**n = 

The first five Fibonacci matrices will be:



Using the above definitions, investigate the following. You will need to show some examples to support your conclusions.

**1.** The addition of two consecutive Fibonacci matrices. [3 marks]

**2.** The product of two consecutive Fibonacci matrices [3 marks]

**3.** The product of two non-consecutive Fibonacci matrices [3 marks]

**4.** The value of the determinant of Fibonacci matrices [3 marks]

**5.** Solutions of simultaneous equations of the form

fnx + fn+1y = fm

fn+1x + fn+2y = fm+1

such that m ≥ n + 1.

eg f4x + f5y = f9 gives 2x + 3y = 21

f5x + f6y = f10 3x + 5y = 34

or

f6x + f7y = f7 gives 5x + 8y = 8

f7x + f8y = f8 8x + 13y = 13

but not

f9x + f10y = f5 gives 21x + 34y = 3

f10x + f11y = f6 34x + 55y = 5

[4 marks]

**TOTAL 16 marks**